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Introduction to High Temperature Plasma Physics

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Introduction to high temperature plasma physics

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Most of the matter in the Universe is in the plasma state. A plasma will be defined and the concepts of quasi-neutrality and Debye distance introduced. The subject has its historical roots in gas discharge, astrophysics and ionospheric physics. Key theoretical concepts were developed in the context of those subjects. However, only in the last two decades, under the pressures of the controlled thermonuclear and space exploration programmes, have these concepts been tested experimentally. The theory of collisions in plasma has special features owing to the Coulomb nature of the interaction. Magnetized plasma is a medium in which a rich variety of small signal waves can propagate. Charged particle–wave interactions lead to collisionless (Landau) damping and growth mechanisms. Finally, nonlinear phenomena in plasma can and do change its transport properties by orders of magnitude. Our lack of detailed understanding of these nonlinear phenomena applies equally to natural plasmas and to plasmas in both inertially and magnetically confined fusion systems. This feature provides the challenge and the fascination of high temperature plasma physics.

Introduction

High temperature plasma physics deals with the behaviour of highly ionized gases (Spitzer 1956). In principle the subject has a stark simplicity since it covers the interaction of structureless charged particles obeying well known physical laws. It is therefore paradoxical that the behaviour of an ionized gas is frequently unpredictable. The subject has its origins in gas discharge physics (Thomson 1906; Townsend 1915; von Engel & Steenbeck 1932), ionospheric physics (Appleton & Barnett 1925; Ratcliffe 1959) and astrophysics (Alfvèn 1950). Several key theoretical concepts were developed in connection with these earlier fields; these have been extended and extensively tested experimentally in connection with the thermonuclear fusion and space programmes.

In this paper I will outline the structure of the subject, emphasize fundamental concepts and their experimental tests, and finally explain why plasma remains an unpredictable state of matter.

Debye distance

One key concept is that of Debye distance. If a region of ionized gas is considered in which there are only particles of one charge sign, then from Poisson's law the electric potential ϕ will vary in a one-dimensional case according to the equation

$$\frac{d^2\phi}{dx^2} = \frac{n_e e}{\epsilon_0}, \tag{1}$$

where n_e is the electron number density, e is the electronic charge and ϵ_0 is the dielectric constant of free space. The change in energy, $\Delta\epsilon$, of a particle crossing this region of thickness λ_D is

$$\Delta\epsilon = n_e e^2 \lambda_D{}^2/2\epsilon_0. \tag{2}$$

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Equating $\Delta\epsilon$ to the mean thermal energy $(\frac{1}{2}T_e)$ of the light, highly mobile electrons gives

$$\lambda_D = (\epsilon_0 T_e/n_e e^2)^{\frac{1}{2}}. \tag{3}$$

The 'Debye distance' λ_D is therefore a measure of the scale length over which the densities of electron and ion charges can be sensibly different. It is also the distance beyond which at any instant the electric field due to a particular particle is shielded by its neighbours. Debye & Hückel (1923) derived the result in connection with a charge immersed in an electrolyte.

An ionized gas with linear dimension L such that $L \gg \lambda_D$ is termed a plasma (Langmuir 1929). In such a plasma there must be quasi-neutrality, that is, nearly equal numbers of positive and negative charges to avoid excessive electrostatic fields.

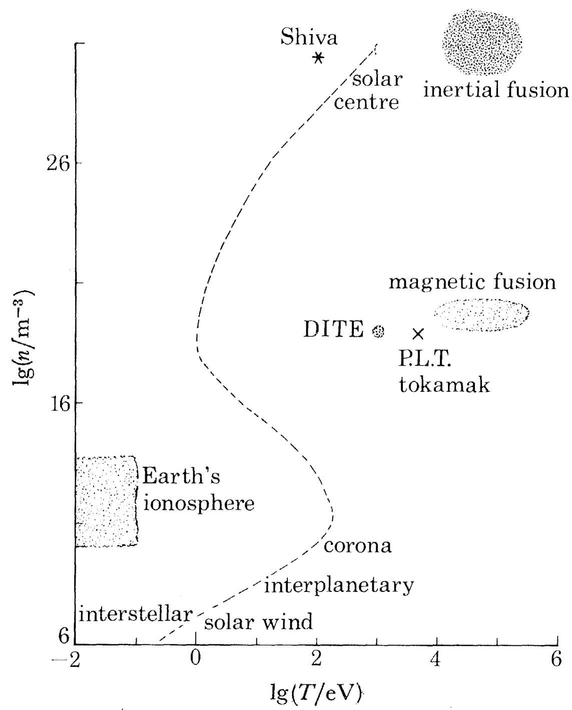

FIGURE 1. Natural- and laboratory-plasma parameters. ---, Conditions in the Sun from the core to the solar wind. Shiva is a laser fusion experiment. DITE and P.L.T. are tokamaks. (After C. J. H. Watson 1974 In *Plasma physics*. London: The Institute of Physics.)

Figure 1 shows the density and temperature of some important naturally occurring and laboratory plasmas. One distinction between space and laboratory plasma is that in the former the scale length of observation (for example, by satellite) is typically smaller than the Debye distance whereas the opposite is true in the laboratory. Hence space observations give local particle distributions but are weak on macroscopic quantities sch as currents and overall field geometries (Alfvèn 1979). It follows that there is much to be gained by combining information from both fields.

Coulomb collisions

The forces between charged particles obey the inverse square law and so are of long range, in comparison with those for collisions between neutral atoms. The Rutherford cross section for the scattering of one charge particle by another through an angle θ is

$$\sigma(\theta) = (q_1 q_2)^2 (m_1+m_2)^2/(8\pi\epsilon_0 m_1 m_2 u^2 \sin^2 \tfrac{1}{2}\theta)^2, \tag{4}$$

where q_1, q_2 are the charges and m_1, m_2 are the respective masses of the two particles, and u is their relative velocity. If one particle is considered and integration is attempted to obtain the effect of its interaction with all other particles, the resultant integral diverges. But since the electric field due to an individual particle is Debye shielded by its neighbours to give an effective electric potential

$$\phi = (q/\epsilon_0 r)\exp(-r/\lambda_D), \tag{5}$$

the integral can be cut off for impact parameters greater than the Debye distance. This procedure leads to a cross-section σ_{ei}, for example, for 90° scattering of electrons by ions:

$$\sigma_{ei} = z_i^2 e^4 \ln\Lambda/25\pi\epsilon_0^2 T_e^2, \tag{6}$$

where p_0 is the distance of closest approach ($p_0 = Z_i e^2/\epsilon_0 T_e$), Z_i is the ion charge and $\Lambda = \lambda_D/p_0$. The Coulomb logarithm is typically *ca.* 10 and expresses the increase in particle scattering due to encounters with 'distant' particles within the Debye sphere. The use of the Debye cut-off corresponds to dividing the electric field at a point into two components. One is due to the uncorrelated motion of particles within the Debye sphere and is included in the collision term. The other is of longer range, and is due to the collective response of the plasma. In a thermal plasma this latter component, corresponding to an equilibrium level of plasma waves, gives a particle scattering *ca.* $1/n\lambda_D^3$ that due to interactions within the Debye sphere. Thus a subsidiary definition of a plasma is that the number of electrons within a Debye sphere should be large. This condition ensures that the plasma can display collective properties and so can be used as a working definition of a 'high temperature' plasma.

The most spectacular demonstration of the reality of the Debye distance and its effect on correlations is provided by experiments on the scattering of light by plasma electrons. Light with an incident wavenumber $\boldsymbol{k}_i$ is detected with a scattered wavenumber $\boldsymbol{k}_s$, corresponding to scattering from density fluctuations with $\boldsymbol{k}_f = \boldsymbol{k}_i - \boldsymbol{k}_s$. The measured spectrum of scattered light depends dramatically on the value of $k_f\lambda_D$. For $k_f\lambda_D > 1$, the (Thomson) scattering is basically from uncorrelated electrons and the spectrum determined by Doppler broadening coming from the electron thermal motion. For $k_f\lambda_D < 1$ the spectrum is now determined by correlated electron motion associated with electron and ion waves in the plasma (Evans & Katzenstein 1969).

Electrical conductivity

The electrical conductivity σ of the plasma can be calculated, given that we have the cross-section σ_{ei} for momentum transfer between electrons and ions. The momentum equation in an electric field E is

$$eE = mV_d\nu_{ei} \tag{7}$$

where V_d is the mean drift velocity of the electrons relative to the ions and ν_{ei} is the collision frequency;

$$\nu = n_i\sigma_{ei}\bar{V}_e = (n_e/Z_i)\,\sigma_{ei}\bar{V}_e \tag{8}$$

while

$$j = n_e eV_d = \sigma E. \tag{9}$$

From these equations we find the electrical conductivity

$$\sigma = 25\pi\epsilon_0^2 T_e^{\frac{3}{2}}/m^{\frac{1}{2}} Z_i e^2 \ln\Lambda. \tag{10}$$

Hence the electrical conductivity is practically *independent* of density and increases rapidly with temperature (Spitzer & Härm 1953). In fact for an electron–proton plasma the conductivity is approximately that of copper when the electron temperature is *ca.* 10^7 K. Experimentally the classical expression for the conductivity is confirmed provided that the ratio of electron drift to thermal velocities is sufficiently small (see, for example, Brown *et al.* 1968).

One special feature of an ionized gas in an electric field is the occurrence of so-called runaway electrons. Electrons whose initial thermal velocities are in the direction of the electric field will be accelerated. Since the scattering cross section is inversely proportional to the square of their energy, the 'mean free path' for these electrons can be increasing with time so fast that in effect they never collide, but 'run away'. The critical energy for runaway, ϵ_{crit}, is given by

$$\epsilon_{\mathrm{crit}} = \tfrac{1}{2}mu^2_{\mathrm{crit}} = (Z_{\mathrm{i}}\,e^3n_{\mathrm{e}}/4\pi\epsilon_0{}^2E)\ln\Lambda. \tag{11}$$

Under restricted conditions corresponding to weak runaway the theory is borne out by experiment (see, for example, Knoepfel & Spong 1979).

By working along similar but more complex lines the other transport coefficients such as those for diffusion and thermal conduction can be calculated (classical theory) for a magnetized plasma (Chapman & Cowling 1960; Braginski 1966). Note that as the plasma temperature increases so, at constant density, the mean free path increases and may become very long compared with the size of the system. In such cases transport coefficients are no longer purely local; the particle orbits, which will be related to the geometry of the system, become important. For such cases a more involved 'neo-classical' theory has been developed (Galeev & Sagdeev 1968).

Fluid model of plasma

One simple (ideal magnetohydrodynamic) model of plasma is that of an infinitely conducting fluid with isotropic pressure p. The equations for such a fluid are (Spitzer 1956):

momentum equation,
$$\rho\,\partial\boldsymbol{V}/\partial t = -\nabla p + \boldsymbol{j}\wedge\boldsymbol{B} - \rho\nabla\phi, \tag{12}$$

Ohm's law,
$$\boldsymbol{E} + \boldsymbol{V}\wedge\boldsymbol{B} = 0, \tag{13}$$

Maxwell's equations,
$$\left.\begin{aligned}\nabla\cdot\boldsymbol{B} &= 0,\\ \nabla\wedge\boldsymbol{E} &= -\partial\boldsymbol{B}/\partial t,\\ \nabla\wedge\boldsymbol{B} &= \boldsymbol{j},\end{aligned}\right\} \quad (15),\ (16)$$

where ρ is the mass density, $\boldsymbol{V}$ is the mean velocity of the fluid and ϕ is the gravitational potential.

From (12) we can see the three distinct possibilities for the local existence of a plasma with its pressure p falling off at the boundaries of the system (i.e. a 'contained' plasma):

(i) gravitational containment,
$$\nabla p + \rho\nabla\phi = 0; \tag{17}$$

this is only feasible in objects of stellar size where the required gravitational potential can be developed;

(ii) inertial confinement,
$$-\nabla p = \rho\,\partial\boldsymbol{V}/\partial t, \tag{18}$$

a transient equilibrium with a lifetime of the order of the transit time of sound across the system; this forms the basis of nuclear weapons and of inertial confinement fusion whether driven by laser, electron, or heavy- or light-ion beams;

(iii) magnetic confinement,
$$\nabla p = \boldsymbol{j}\wedge\boldsymbol{B}, \tag{19}$$

which forms the basis of magnetic confinement fusion experiments, examples being the tokamak, stellarator and pinch systems. Note that we require constant pressure surfaces that are closed in space, and therefore by examination of (19) we require a magnetic field everywhere tangential to such a surface and nowhere equal to zero. By a theorem of Poincaré only a topologically *toroidal* surface can have these properties, given that $\boldsymbol{B}$ is divergence-free. Such a toroidal constant pressure surface is therefore also a *magnetic surface*, the magnetic field everywhere lying in the surface and a given field line exploring in general the whole surface (Morozov & Solov'ev 1966; Kruskal & Kulsrud 1958).

Note that this discussion of magnetic confinement refers only to the case of isotropic pressure. With anisotropic pressure other equilibria are possible with magnetic field lines that leave the plasma volume. Magnetic mirror traps and the van Allen radiation belts are examples of such open systems.

Frozen-in fields

Combination of equations (13) and (15) leads to the result

$$\partial \boldsymbol{B}/\partial t = \nabla \wedge (\boldsymbol{V} \wedge \boldsymbol{B}). \tag{20}$$

This implies that the field changes are the same as if the field lines move with the plasma, since equation (20) is the condition that the total magnetic induction is constant through a circuit, each point of which moves with the local velocity $\boldsymbol{V}$. Thus we have the concept, first enunciated by Alfvèn (1942), that the lines of force are frozen into the plasma. Note that since the plasma velocity at any point is single valued, it is not possible for any motion of the fluid to change the topology of the magnetic field.

Symmetric toroidal hydromagnetic equilibria

To obtain a symmetric solution of the equilibrium equation (19) with $\nabla \cdot \boldsymbol{j} = 0$ it is necessary to have two components of magnetic field in the meridional plane: a poloidal component produced by a toroidal current in the plasma and a component parallel to the major axis of the toroid produced by currents in external conductors (Shafranov 1966). The addition of a toroidal component of field increases the range of possible equilibria. The required poloidal field may also be produced by currents in material conductors *inside* the plasma (see Riviere *et al.*, this symposium). In this case it is not essential for equilibrium to have the field component parallel to the major axis.

Asymmetric hydromagnetic equilibria

There is a wide class of non-symmetric, topologically toroidal systems in which *vacuum* magnetic surfaces are generated entirely by currents in external conductors. These are stellarators and their practical effectiveness in producing particle containment has been directly demonstrated by an experiment in which β-particles made *ca.* 10^7 transits around the torus (Gibson *et al.* 1968).

Ideal m.h.d. stability analysis

Given a magnetostatic equilibrium we can test it for stability. In other words if the plasma is locally displaced by $\xi(r, t)$ we ask whether there are solutions in which ξ increases with time. The linearized theory with which to tackle this question is now very highly developed for the case of small perturbations (Bernstein *et al.* 1958; Hastie, this symposium). In principle it is now possible to answer the question for symmetric toroidal systems; in practice it requires elaborate computing and this may have its own difficulties. General results can be discerned. A rapid rate of change along the pressure gradient of the *direction* of the magnetic field (shear) or an increase outwards in the average magnetic field strength (magnetic well) are stabilizing features.

Non-ideal m.h.d. theory

Various departures from the ideal m.h.d. model can be introduced, such as viscosity and finite resistivity. The latter leads to two important effects. First any equilibrium is a dynamic one, the plasmas diffuse across the magnetic field and can only be maintained in steady state by suitable sources of particles and energy. Secondly, changes in topology of the magnetic field are now

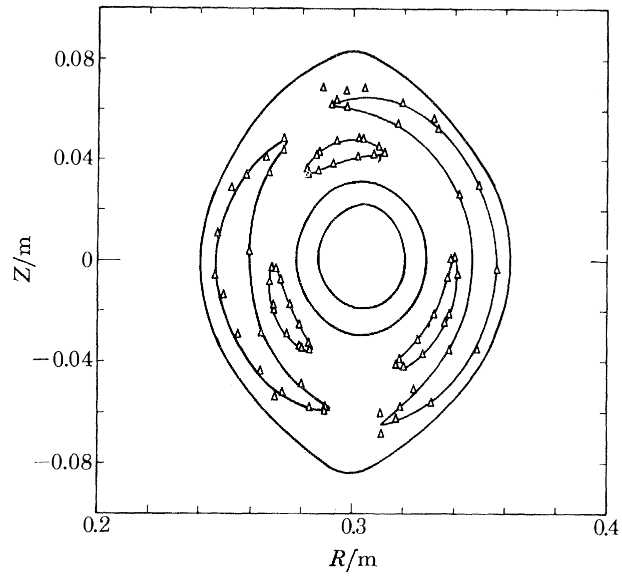

Figure 2. Magnetic surfaces computed for the Tosca tokamak on the basis of external field measurements. The formation of $m = 2$ and $m = 3$ magnetic island structures are illustrated. (From D. C. Robinson *et al.* 1980 *8th International Conference on Plasma Physics and Controlled Nuclear Fusion, Brussels.*)

possible since plasma and field lines are no longer completely frozen together. 'Resistive' instabilities occur, the general properties of which were established in a classic paper by Furth *et al.* (1963). One class of (tearing) modes involves the cutting and reconnection of magnetic field lines with consequent release of magnetic energy in a manner first proposed by Dungey (1958) in the context of the magnetosphere and of solar flares. In the tokamak case these tearing modes lead to the formation of 'magnetic islands', centred on 'rational' magnetic surfaces where field lines pass m times around the major axis in going n times round the minor axis of the toroid, m and n being integers. Figure 2 shows an example of such a structure deduced from external field measurements. If islands grow on neighbouring surfaces of different rationality, m/n, then if they overlap they can lead to a complete break-up of the magnetic surfaces, field lines exploring the whole volume in an ergodic way. This may be the mechanism of the so-called disruptive instability of tokamaks in which the magnetic energy of the plasma current is suddenly and explosively converted into plasma thermal energy and deposited on the wall. Similar phenomena apparently occur in solar flares (Brown & Smith 1980).

An m.h.d. stability question of major interest to the fusion programme is the limiting ratio of plasma pressure to magnetic field pressure, $\beta\ (= 2\mu_0 nT/B^2)$. When this ratio is large enough the confined plasma is able to drive unstable modes which involve distortions of the magnetic field. Early indications are that plasma confinement remains unchanged even after the theoretical threshold for such (fine scale) instabilities is crossed.

Two-fluid theory

The plasma model can be extended by treating it as two fluids, one of electrons and one of ions. By retaining inertial terms high frequency phenomena can be included such as that due to the gyration of electrons about the field lines. With this model the propagation of small signal waves in an infinite, uniform magnetized plasma can be studied. Six separate dispersion branches are found; figure 3 shows a typical frequency, ω, against wave-number, k, plot. Two upper branches correspond to the propagation of two polarizations of electromagnetic waves as affected by the oscillating conduction currents due to the motion of the plasma electrons in the electric field of

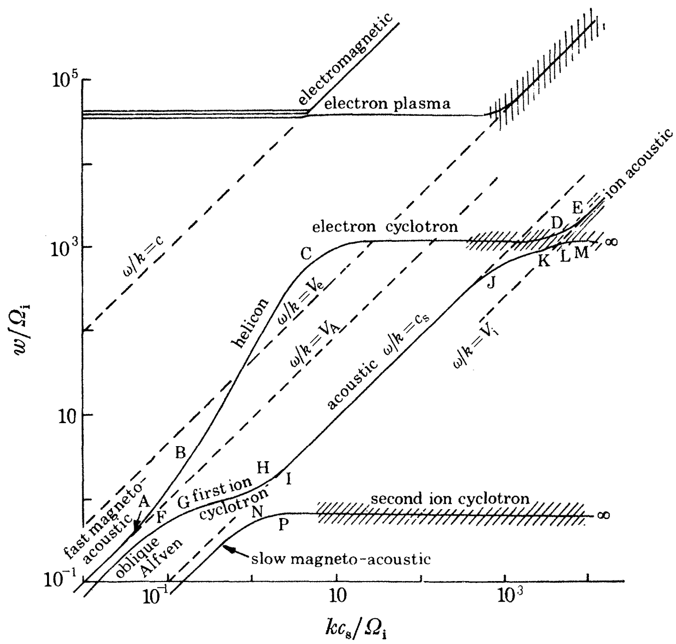

Figure 3. Dispersion diagram for waves in a low β-hydrogen plasma propagating at 45° to the magnetic field. (From T. E. Stringer 1963 *Plasma Phys.* **5**, 89 and J. J. Sanderson 1974 In *Plasma physics*. London: The Institute of Physics.)

the waves. In addition there is a longitudinal electrostatic branch corresponding to the (Langmuir) oscillations of electrons in the space charge fields resulting from their displacement, the frequency is approximately the Langmuir or plasma frequency, ω_p, where

$$\omega_p{}^2 = n_e e^2/\epsilon_0 m.$$

At lower frequencies there are two branches corresponding to shear and compressional Alfvèn waves. These can be regarded as fluid motions with the magnetic field frozen to the fluid and hence a major contribution from magnetic stresses to the restoring forces. The phase velocities are of the order of the Alfvèn speed V_A:

$$V_A = (B_0{}^2/\mu_0 n_i M_i)^{\frac{1}{2}}.$$

Finally at still lower frequency there is a longitudinal acoustic branch which does not involve significant perturbations of the magnetic field and hence has a phase velocity c_s given by

$$c_s \sim (T_e/M_i)^{\frac{1}{2}},$$

where the inertia comes from the ions and the restoring force from the electron pressure. Note that not only are there six branches but also that their propagation properties are anisotropic, i.e. depending strongly on the direction of propagation relative to the magnetic field direction.

If the plasma is inhomogeneous, with density, temperature or magnetic field gradients, then the number of branches is unchanged but they are modified in ways which are important in relation to the stability of the plasma.

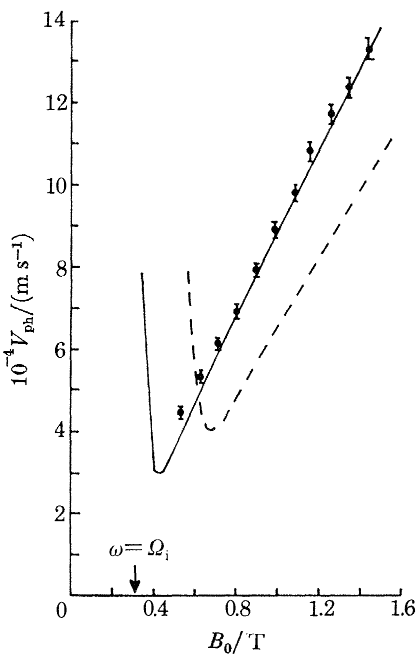

FIGURE 4. Measured phase velocity of shear Alfvèn waves. —, Theoretical, taking into account collisions between ions and neutral atoms; $n_i = 10^{21}\ m^{-3}$; $n_0 = 1.5 \times 10^{20}\ m^{-3}$. – – –, Experimental, $n_i = 10^{21}\ m^{-3}$; $n_0 = 8 \times 10^{20}\ m^{-3}$. (From D. F. Jephcott & P. M. Stocker 1962 *J. Fluid Mech.* **13**, 587.)

Experiments on small amplitude waves

In the 1960s and early 1970s there was an intensive programme of experiments to test in the laboratory the wave propagation properties of plasma. Figures 4–6 show sample results for the shear Alfvèn, compressional Alfvèn and acoustic waves respectively. A very wide range of experiments show in general excellent agreement with the theory including resonances, cut-offs and damping mechanisms.

Kinetic model

The kinetic model of plasma is a further refinement which takes into account the distribution of velocities of particles in the plasma. The particle distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ is, after Liouville's theorem, constant along the free trajectory of a particle in the absence of collisions. This leads to the Vlasov or collisionless Boltzmann equation which gives the evolution of the distribution function under the influence of the electric and magnetic fields in the plasma (Vlasov 1938):

$$\frac{\partial f_j}{\partial t} + \boldsymbol{V}\cdot\nabla f_j + q_j\,\frac{\boldsymbol{E}+\boldsymbol{V}\wedge\boldsymbol{B}}{M_j}\,\nabla_{\mathrm{V}} f_j = 0.$$

The most important consequence of the kinetic theory is collisionless or Landau damping of waves. For example consider an electrostatic wave propagating in a plasma with a Maxwellian velocity distribution. Because of their thermal motion, particles 'see' the wave at Doppler shifted frequencies ω^1 where,

$$\omega^1 = \omega - \boldsymbol{k}\cdot\boldsymbol{V}.$$

Hence some 'resonant' particles see the wave at zero frequency. Particles moving slightly slower than the wave will be accelerated by the wave field and vice versa. Because for a one-dimensional Maxwellian distribution there will always be more particles going slower than the wave than those going faster, there will be a net extraction of energy from, and hence a damping of, the wave.

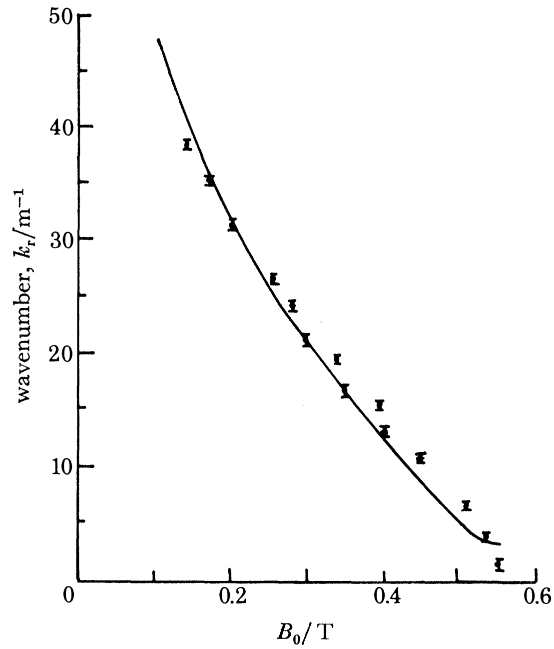

FIGURE 5. Measured wavenumber against magnetic field for fixed frequency compressional Alfvèn wave. Theoretical prediction. (From D. F. Jephcott & A. Malein 1964 *Proc. R. Soc. Lond.* A **278**, 243.)

Much theoretical attention has been given to understanding this apparently collisionless, or Landau, damping, named after its discoverer (Landau 1946). In fact in the complete absence of collisions there is no dissipation, the information is stored in the modified distribution function and can even be recovered by rather special echo experiments (Gould *et al.* 1967). However the information is stored on a fine scale so that in many situations small-angle Coulomb collisions are sufficient to restore the distribution function to a Maxwellian and hence to provide genuine damping. By the same arguments if, because of plasma inhomogeneity or because electric current flows, the reduced distribution function has $(\partial f/\partial u)_{u=\omega/k} > 0$ at the phase velocity of a longitudinal wave, then there may be 'inverse Landau damping' or, in other words, temporal growth of the wave amplitude (Penrose 1960). The reduced distribution function f is

$$f = f_{\mathrm{e}} + (m_{\mathrm{e}}/M_{\mathrm{i}})f_{\mathrm{i}}.$$

The phenomenon of Landau damping and growth is at the heart of the theory of micro-instabilities with which the stability of small signal waves is studied. Landau damping is also the basis of most methods of heating plasmas by high frequency fields.

The first definitive experimental demonstration of Landau damping was given by Malmberg & Wharton (1966). Figure 7 shows the excellent agreement between their data and the theory.

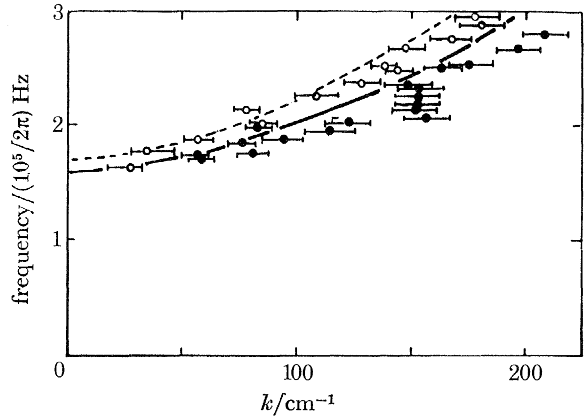

FIGURE 6. Dispersion curves for propagation of ion acoustic waves in a mercury positive column with a current of 9 A in an applied longitudinal field of 45×10^{-4} T, $p = 5 \times 10^{-2}$ Pa. (From P. F. Little 1962 *Nature, Lond.* **194**, 1137.)

NONLINEAR EFFECTS

Since, almost without exception, laboratory and natural plasmas are linearly unstable it is vitally important to understand and to be able to predict the nonlinear consequences.

Some progress has been made for the case of micro-instabilities. The so-called quasi-linear theory has been developed (Drummond & Pines 1962; Vedenov *et al.* 1962) for the case where a spectrum of non-interacting waves, growing owing to inverse Landau damping, causes a change

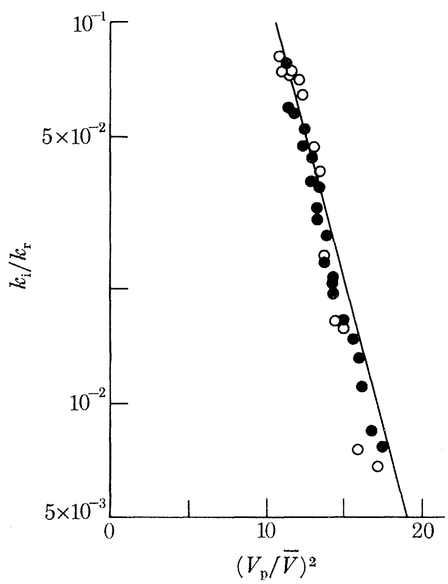

FIGURE 7. Ratio of imaginary and real parts of the wavenumber as a function of the ratio between the phase velocity of the plasma wave and the random speed of electrons: ●, $KT = 6.5$ eV; ○, $KT = 9.6$ eV. ——, Prediction of the Landau damping theory. (From Malmberg & Wharton 1966.)

in the velocity distribution, which by reducing $(\partial f/\partial u)_{u=\omega/k}$ eventually turns off the instability. The theory has been criticized for being poorly founded and vague (Cook 1974). Nevertheless its predictions have been confirmed by very careful experiments designed to satisfy the possibly special conditions for the validity of the theory (Roberson & Gentle 1971).

Apart from interacting with the particles, waves will also in general interact with each other (because of the nonlinearity of the third term in the Vlasov equation). Considerable success has been achieved in a wide range of experiments on nonlinear wave–wave and wave–particle interactions (Franklin 1977). This work deals with reversible interactions between coherent waves and has given good agreement with theory. It provides some of the basic information for a weak turbulence theory in which the plasma is treated as an assembly of interacting waves and particles (Kadomtsev 1965). This coherent-wave work is analogous to the measurement of elastic collision cross sections, reversible processes to be included in an eventual kinetic theory of irreversible phenomena.

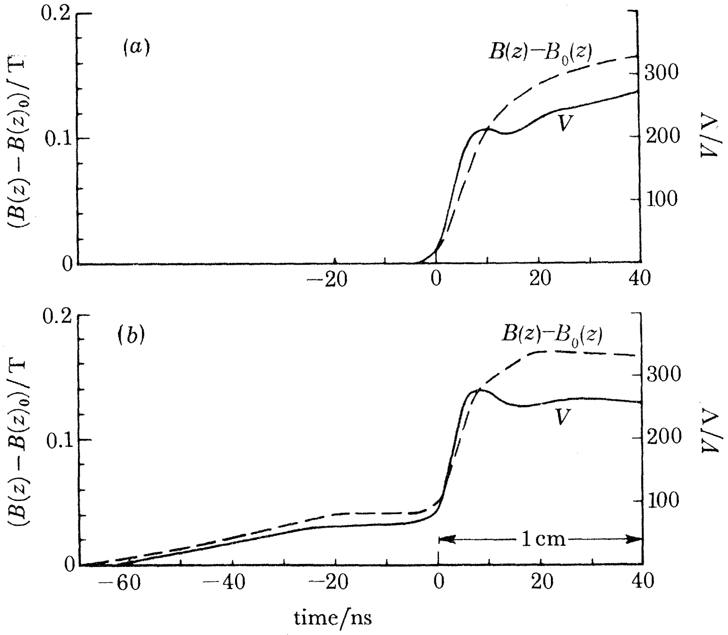

FIGURE 8. Variation with time of magnetic field and electric potential at a fixed point when shock wave passes. Shock wave propagating perpendicular to magnetic field with quoted Alfvèn Mach numbers. (*a*) $M = 2.5$, $B_0(z) = 0.12$ T, $V_s = 2.4\times10^7$ cm s^{-1}; (*b*) $M = 3.7$, $B_0(z) = 0.075$ T, $V_s = 2.5\times10^7$ cm s^{-1}; where M is the Alfvèn Mach number, $B_0(z)$ is the initial magnetic field strength and V_s the shock speed. (From Paul *et al.* 1967.)

The basic purpose of a plasma turbulence theory is to predict the 'anomalous' transport properties of the plasma (Kadomtsev 1965). Only very limited success has so far been achieved. The most commonly quoted result is that the effective coefficient for diffusion across a magnetic field, D_{eff}, resulting from a micro-instability of growth rate γ and perpendicular (to B) wave-number $k_\perp$ is

$$D_{\text{eff}} \sim \gamma/k_\perp^2.$$

The derivation of this result (Kadomtsev 1965) is not entirely convincing and there is no wide range of experimental results against which it can be tested (but see Riviere *et al.*, this symposium).

Experimentally it is found that transport coefficients *are* increased by orders of magnitude owing to micro-instabilities. An extreme nonlinear case is a shock wave. Figure 8 shows the variation of magnetic field strength with time as a shock wave passes through a magnetized plasma From the thickness of the shock wave it can be shown that the electrical resistivity of the plasma is increased by a factor of about 10 over the classical value (Paul *et al.* 1965). A series of light-scattering experiments showed that in this case there is a level of ion-acoustic turbulence in the front sufficient to account for the observed dissipation (Paul *et al.* 1967, 1970). Similarly, figure 9 shows the electrical conductivity of a plasma subjected to a pulsed electric field (Hamberger *et al.* 1968). Again the conductivity is much lower than the classical value and decreases with increasing electric field. Here two régimes may be discerned corresponding to ion-acoustic turbulence at moderate fields and Langmuir turbulence at the highest fields. In these experiments the turbulent fields lead to a complete absence of runaway electrons.

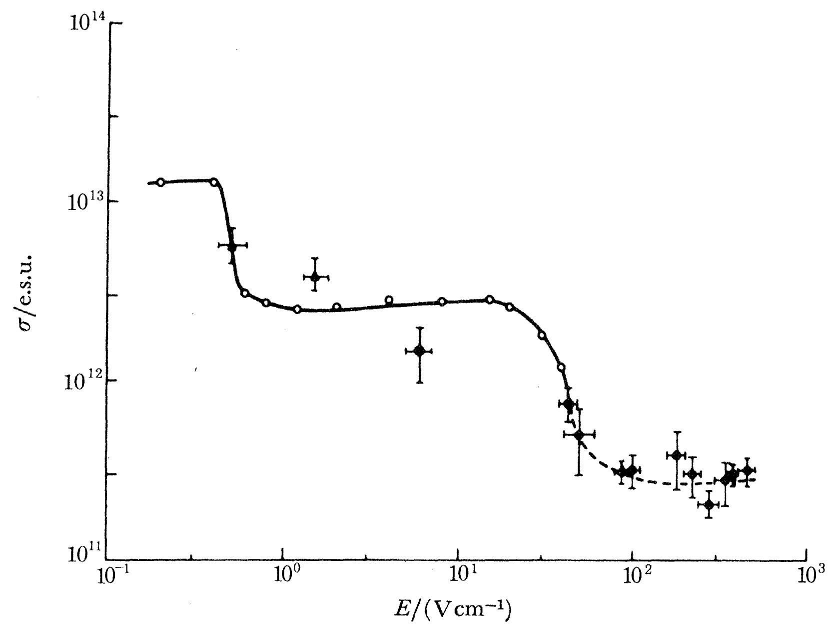

FIGURE 9. Variation of electrical conductivity of a plasma with electric field for hydrogen with electron density $\bar{n}_e \approx 10^{18}\ \mathrm{m^{-3}}$. (From Hamberger *et al.* 1968.) ○, From Demidov *et al.* (1967, *Soviet Phys. Dokl.* **12**, 467); ⊕, Hamberger *et al.* (1968); ⊥, Hamberger *et al.* (1968), from pre-ionization discharge.

In the tokamak the electron thermal conduction along the radius of the discharge is the dominant energy loss and is typically 10^2 times larger than predicted by the neo-classical theory. We do not have enough measurements to determine clearly the relative importance of m.h.d. and micro-instabilities. Both certainly occur. In laser fusion it is ironic that micro-instabilities *reduce* the electron thermal conduction and that this is undesirable since it reduces the efficiency of the ablation process and hence the compression of the core (Kilkenny & Gray 1980).

Anomalous processes are also common in natural plasmas. For example a collisionless shock occurs where the solar wind impinges on the geomagnetic field. Similarly a solar flare involves the rapid conversion of magnetic to thermal energy through instability mechanisms (Brown & Smith 1980).

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Conclusions

There is now a fairly complete theory of a classical plasma, i.e. one in which transport processes are solely due to binary collisions. The key concepts of this theory such as Debye screening, Coulomb collision rates, Alfvèn waves, Landau damping, and linear wave propagation are well demonstrated experimentally. The classical theory is tediously complex, but it is in principle capable of predicting completely the behaviour of a particular system, taking into account the geometry, the collision processes, and the equilibrium and evolutionary equations. The predictions of such a theory are, however, commonly wrong since almost all plasmas are linearly unstable and their behaviour is determined by the equilibrium level of the fluctuations that develop. (One notes that there is fairly good agreement with the linear stability theory in that when instability is predicted fluctuations are seen.) However the nonlinear consequences of instability are not well predicted by current theory. In certain simple cases, such as anomalous resistivity we do now have enough experimental and theoretical information to enable confident predictions to be made. The similarity laws for plasma can be used to scale one situation to another in such cases where one mechanism is dominant (Connor & Taylor 1977; Schindler 1968). However in many situations, the tokamak being the most studied example, we have several nonlinear processes occurring simultaneously and possibly interactively in the same plasma. In such cases we are currently reduced to the use of empirical scaling laws obtained by regression analysis of existing data (Hugill & Sheffield 1978). In the future we can expect to make progress in the understanding of these situations through intensive measurements of existing plasmas. In particular, measurements of fluctuating quantities would enable us to identify the main mechanisms and instabilities responsible for the plasma properties. A major bottleneck here is the lack of methods for measuring fluctuating electric and magnetic fields in the interior of laboratory plasmas. Purely empirically, there will also be continuous progress through the extension of the parameter range of the plasmas under study.

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